2022 April 5

Basic Concepts of Graph Theory

1 | THIS IS WHAT A MATH PAPER LOOKS LIKE IN THE *JOURNAL OF GRAPH THEORY*!

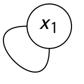
Dear Richard,

I do enjoy a good hand-written letter, but this time I'm going to explore typesetting for maximum readability. I will do my best to have this still read like a letter from me to you—which it still very much is. I made this document using a markup programming language called LaTeX, pronounced "la-tech" or "lay-tech". It was designed specifically for typesetting mathematical notation to allow for easier sharing of ideas. Over the years, many people have worked on extending its support for areas of math that require diagrams for best discussion, such as graph theory. In particular, the format of this document is basically what you would see in an issue of *Journal of Graph Theory*, which is one of the leading publications in the field. I'm not very practiced in typesetting graphs yet, but I look forward to improving, and I'll take care not to send you anything that doesn't make at least basic sense!

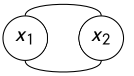
1.1 | What is—and isn't—a graph

In my last letter, I introduced the basic notion of a graph, but there are some details that I need to pin down for you to exactly specify the type of object we're talking about when we discuss graphs and which objects we're *not* talking about. As simple as graphs seem, there are some potential subtleties we have to be careful to address in the beginning.

Even though we're basically just talking about dots and line segments between them, within that there's some ambiguity. A couple of examples: Do we consider the following objects to be graphs? (Instead of dots, the standard representation of a node in LaTeX is a circle with the node name inside.)



Graph 1



Graph 2

We say that Graph 1 has a loop and that Graph 2 has multiple edges. Canonically, graph theory doesn't include these types of objects. They're still neat, just non-standard. I'll describe many types of graphs in future letters. For now, I just want to introduce the ideas of: a *path* and then leave you with a problem to think about.

Definition A *path* is a sequence (a countable ordered list) of vertices and edges in a graph such that
(1) The sequence alternates between edges and vertices, starting and ending with vertices; and
(2) Each edge in the sequence joins the vertices that occur immediately before and after it in the sequence.

We say that the *length* of a path is the number of edges in the path, including repetitions when they occur.

Now, an exercise for you: What is the fewest number of state-crossings needed to drive along a path (as defined above) that visits all the states of Tennessee, New Mexico, Louisiana, Arkansas, Colorado, Alabama, Kansas, Missouri, Oklahoma, and Mississippi? I've sent along a map of the U.S. with this letter. Hope it gets to you!

A bonus exercise is to think about this question: What is the least number of colors needed to color the map such that no two states sharing a boundary have the same color?

As you said in your last letter, convolution is sometimes unhelpful when solving problems. In general as we work together, you might start working on problems by first exploring the very simplest situations and then seeing how much complexity you need to add in. This is a doing-math-strategy principle. There are more ways for a situation to be complex, so it takes longer for you, the mathematician, to analyze all the complex situations. There being fewer ways for a situation to be simple, we can go for the low-hanging fruit first and possibly save a whole bunch of time and effort.:)

Until next time,
Julia